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LRS Bianchi Type-I Universe in Barber's Second Self Creation Theory

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Abstract We consider Barber's second self creation theory with perfect fluid source for an LRS Bianchi type-I metric by using deceleration parameter to be time dependent where the metric potentials are taken as function of x and t. The present models are free from singularity and the results are consistent within the observational limit. Some physical properties of the models are also discussed.

Keywords Exact solutions · Barber's self creation theory · LRS Bianchi type models

1 Introduction

Several modification of Einstein's general relativity have been proposed and extensively studied so far by many cosmologists to unify gravitation and many other effects in the universe. Barber [1] produced two continuous self-creation theories by modifying the Brans-Dicke theory and general relativity. The modified theories create the universe out of self-contained gravitational and matter fields. Brans and Dicke [3] theory develops Mach's principle in a relativistic framework by assuming interaction of inertial masses of fundamental particles with some cosmic scalar field coupled with the large scale distribution of matter in motion. However, Barber [1] has included continuous creation of matter in these theories.

The universe is seen to be created out of self-contained gravitational, scalar and matter fields. However, the solution of the one-body problems reveals unsatisfactory characteristics of the first theory and in particular the principle of equivalence is severely violated. The first theory can not be derived from an action principle. Brans [2] has pointed out that Barber's first theory is in disagreement with experiment as well as inconsistent, in general, since the

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equivalent principle is violated. The second theory retains the attractive features of the first theory and overcomes previous objections. These modified theories create the universe out of self-contained gravitational and matter fields. In Barber's second self-creation theory, the gravitational coupling of the Einstein field equations is allowed to be a variable scalar on the space-time manifold. This second theory is a modification of general relativity to a variable G-theory and predicts local effects, and secondly is an adaptation of general relativity to include continuous creation and is within the observational ambit. In this theory the Newtonian gravitational parameter G is not a constant but a function of time. Further, the scalar field does not gravitate directly, but simply divide the matter tensor, acting as a reciprocal gravitational constant. It is postulated that this scalar field couples with the trace of energy momentum tensor. Moreover, this theory is capable of verification or falsification. It can be done by observing the behaviour of both bodies of degenerate matter and photons. An observation of anomalous precession in the orbits of pulsars about central masses and an accurate determination of the deflection of light and radio waves passing close to the sun would verify or falsify such a theory and determines λ. The theory predicts the same precession of the perihelia of the planets as general relativity and in that respect agrees with observation to within 1%. In the limit $\lambda \to 0$ this theory approaches the Einstein's theory in every respect. In view of the consistency of Barber's second theory of gravitation, we intend to investigate some of the aspects of this theory in this paper.

Several cosmologists have studied various aspects of Robertson-Walker model in Barber's second self-creation cosmology with perfect fluid satisfying the equation of state $p = (\gamma - 1)\rho$, where $1 \leq \gamma \leq 2$. Pimentel [18] and Soleng [41] have discussed the Robertson-Walker solutions in Baber's second self-creation theory of gravitation by using power law relation between the expansion factor of the universe and the scalar field. Carvalho [6] studied a homogeneous and isotropic model of the early universe in which parameter gamma of 'gamma law' equation of state varies continuously with cosmological time and presented a unified description of early universe for inflationary period and radiation- dominating era. Singh and Singh [39], Reddy [30, 31], and Reddy et al. [33] have presented Bianchi type space-times solutions in Barber's second theory of gravitation. Reddy and Venkateswarlu [32] presented Bianchi type-VI_0 cosmological solutions in Barber's second theory of gravitation both, in vacuum as well as in the presence of perfect fluid with pressure equal to energy density. Shanthi and Rao [36] studied Bianchi type II and III space-times in second theory of gravitation, both in vacuum as well as in presence of stiff-fluid. Ram and Singh [28, 29] have discussed spatially homogeneous and isotropic R–W and Bianchi type-II models of the universe in Barber's second self-creation theory of gravitation in the presence of perfect fluid by using 'gamma-law' equation of state. Panigrahi and Sahu [16] have presented plane symmetric cosmological micro model in this modified theory of Einstein's general relativity.

The Einstein's field equations are coupled system of high non-linear differential equations and we seek physical solutions to the field equations for their applications in cosmology and astrophysics. In order to solve the field equations we normally assume a form for the matter content or that space time admits killing vector symmetries [10]. Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter which was proposed by Berman [4]. In simple cases the Hubble law yields a constant value of deceleration parameter. It is worth observing that most of the well-known models of Einstein's theory and Brans-Deke theory with curvature parameter $k = 0$, including inflationary models, are models with constant deceleration parameter. In earlier literature cosmological models with a constant deceleration parameter have been studied by Berman [4], Berman and Gomide [5], Johri and Desikan [8], Singh and Desikan [38], Maharaj and Naidoo [12], Pradhan et al. [19, 25] and others. But redshift magnitude test has had a chequered history. During the 1960s and the 1970s, it was used to draw very categorical conclusions. The deceleration parameter q_0 was then claimed to lie between 0 and 1 and thus it was claimed that the universe is decelerating. Today's situation, we feel, is hardly different. Observations [9, 34] of Type Ia Supernovae (SNe) allow to probe the expansion history of the universe. The main conclusion of these observations is that the expansion of the universe is accelerating. So we can consider the cosmological models with variable deceleration parameter. The readers are advised to see the papers by Vishwakarma and Narlikar [45] and Virey et al. [42] and references therein for a review on the determination of the deceleration parameter from Supernovae data. Recently Pradhan and Otarod [20, 21] have obtained the solution of Einstein's field equations with time dependent deceleration parameter and Λ-term in presence of perfect and bulk viscous fluid.

In recent years, Mohanty et al. [14], Panigrahi and Sahu [17], Sahu and Mohanty [35], Venkateswarlu and Kumar [43], Singh and Kumar [37], Singh et al. [40], Venkateswarlu et al. [44] have studied Barber's second self-creation theory of gravitation in various contexts. For simplification and description of large scale behaviour of the actual universe, locally rotationally symmetric [henceforth referred as LRS] Bianchi I space-time have widely studied [7, 11, 13, 17, 22–24, 26, 27]. In this paper we obtain cosmological solutions for an LRS Bianchi type-I metric in Barber's second self-creation theory of gravitation with perfect fluid for the time dependent deceleration parameter models of the universe.

2 The Metric and Field Equations

We consider LRS Bianchi type-I space-time

$$ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2), \tag{1}$$

where the metric potentials A and B are functions of x and t. The field equations in Barber's second self-creation theory of gravitation are given by [1]

$$R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi T_{ij}}{\phi} \tag{2}$$

and

$$\Box\phi = \frac{4\pi}{3}\lambda T, \tag{3}$$

where $\Box\phi \equiv \phi^{;k}_{;k}$ is the invariant d'Alemberian and T is the trace of the energy momentum tensor that describes all non-gravitational and non-scalar field matter and energy. λ is a coupling constant to be determined from the experiments. The measurments of the deflection of light restrict the value of coupling to $|\lambda| \le 0.1$. In the limit $\lambda \to 0$ this theory approaches the standard general relativity theory in every respect and $G = \frac{1}{\phi}$.

The energy momentum tensor T_{ij} for a perfect fluid distribution is given by

$$T_{ij} = (p + \rho) u_i u_j - p g_{ij}, \tag{4}$$

where p is the pressure, ρ the energy density and u_i represents the four velocity vector. Corresponding to metric (1), the four velocity vector u_i satisfies the equation

$$g_{ij} u^i u^j = 1. \tag{5}$$

The Bianchi identities in contravariant form applied to (2) are

$$W_{;i}T^{ij} + WT^{ij}_{;i} = 0, \tag{6}$$

where $W = -8\pi\phi^{-1}$ and in general relativity $W = -8\pi G$. A comma and a semicolon denote ordinary and covariant differentiation, respectively. In a comoving coordinate system, the surviving components of the field (2)–(6) for metric (1) are

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{B'^2}{A^2B^2} = -8\pi\phi^{-1}p, \tag{7}$$

$$\dot{B}' - \frac{B'\dot{A}}{A} = 0, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{B''}{A^2B} + \frac{A'B'}{A^3B} = -8\pi\phi^{-1}p, \tag{9}$$

$$\frac{2B''}{A^2B} - \frac{2A'B'}{A^3B} + \frac{B'^2}{A^2B^2} - \frac{2\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} = 8\pi\phi^{-1}\rho, \tag{10}$$

$$\ddot{\phi} + \frac{\dot{A}\dot{\phi}}{A} + \frac{2\dot{B}\dot{\phi}}{B} + \frac{A'\phi'}{A^3} - \frac{2B'\phi'}{A^2B} - \frac{\phi''}{A} = (\rho - 3p)\left(\frac{8\pi\lambda}{3}\right). \tag{11}$$

Here and in what follows a prime and a dot indicate partial differentiation with respect to x and t respectively.

3 Solutions of the Field Equations

Equation (8), after integration, yields

$$A = \frac{B'}{l}, \tag{12}$$

where l is an arbitrary function of x.

Equations (7) and (8), with the use of (12), reduce to

$$\frac{B}{B'}\frac{d}{dx}\left(\frac{\ddot{B}}{B}\right) + \frac{\dot{B}}{B'}\frac{d}{dt}\left(\frac{B'}{B}\right) + \frac{l^2}{B^2}\left(1 - \frac{Bl'}{B'l}\right) = 0. \tag{13}$$

If we assume $\frac{B'}{B}$ as a function of x alone, then A and B are separable in x and t, (13) gives after integration

$$B = lS(t), \tag{14}$$

where $S(t)$ is an arbitrary function of t.

With the help of (14), (12) becomes

$$A = \frac{l'}{l}S. \tag{15}$$

Now the metric (1) takes the form

$$ds^2 = dt^2 - S^2(t)[dX^2 + e^{2X}(dy^2 + dz^2)], \tag{16}$$

where $X = \ln l$.

Further, by use of (14) and (15), (7) , (10) and (11) take the form

$$\frac{2\ddot{S}}{S} + \frac{\dot{S}^2}{S^2} - \frac{1}{S^2} = -8\pi\phi^{-1}p, \tag{17}$$

$$\frac{3}{S^2} - \frac{3\dot{S}^2}{S^2} = 8\pi\phi^{-1}\rho, \tag{18}$$

$$\ddot{\phi} + \frac{3\dot{\phi}\dot{S}}{S} - \frac{3\phi' l}{l' S^2} - \frac{l^2}{l' S^2}\frac{d}{dx}\left(\frac{\phi'}{l'}\right) = \frac{8\pi\lambda}{3}(\rho - 3p). \tag{19}$$

It has been assumed that the scalar field (ϕ) depends only on cosmic time t only, and hence (19) with the use of (17) and (18) reduces to

$$\frac{\ddot{\phi}}{\phi} + \frac{3\dot{\phi}\dot{S}}{\phi S} - \frac{2\lambda\ddot{S}}{S} = 0. \tag{20}$$

The function $S(t)$ remains undetermined. To obtain its explicit dependence on t, one may have to introduce additional assumptions. In the following we assume the deceleration parameter to be constant to achieve this objective i.e.

$$q = -\frac{S\ddot{S}}{\dot{S}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) = b \quad \text{(variable)}, \tag{21}$$

where $H = \dot{S}/S$ is the Hubble parameter. The above equation may be rewritten as

$$\frac{\ddot{S}}{S} + b\frac{\dot{S}^2}{S^2} = 0. \tag{22}$$

The general solution of (22) is given by

$$\int e^{\int \frac{b}{S} dS} dS = t + k, \tag{23}$$

where k is an integrating constant.

In order to solve the problem completely, we have to choose $\int \frac{b}{S} dS$ in such a manner so that (23) be integrable.

Let us consider

$$\int \frac{b}{S} dS = \ln L(S), \tag{24}$$

which does not effect the nature of generality of solution. Hence from (23) and (24), one can obtain

$$\int L(S) dS = t + k. \tag{25}$$

Of course the choice of $L(S)$ is quite arbitrary but, since we are looking for physically viable models of the universe consistent with observations, we consider the following cases.

4 Solution in the Exponential Form

Let us consider $L(S)=\frac{1}{k_1 S}$, where k_1 is an arbitrary constant.

In this case, on integrating, (25) gives the exact solution

$$S(t)=k_2 e^{k_1 t}, \tag{26}$$

where $k_2=e^{kk_1}$.

Using (26) in (20) we obtain the expression for Barber's scalar function ϕ as

$$\phi=-c_1 e^{-m_1 k_1 t}-c_2 e^{-m_2 k_1 t}, \tag{27}$$

where $c_1>0$ and $c_2>0$ are arbitrary constants and

$$m_1=\frac{1}{2}(3-\sqrt{9+8\lambda}),$$

$$m_2=\frac{1}{2}(3+\sqrt{9+8\lambda}).$$

As λ is coupling constant to be determined from the experiment ($|\lambda|\le 0.1$), one can suitably choose the value of λ for which $m_1>0$ and $m_2>0$. The negative scalar field ϕ is the most natural choice for matter creation. This type of negative scalar field is discussed by Reddy [30, 31]. In an other alternative to Einstein's general relativity Narlikar and Padmanabhan [15] have presented the cosmological model based on a negative-energy massless scalar creation field.

Using (26) and (27) in (17) and (18), we obtain the expressions for pressure and energy density as

$$p=-\frac{\phi}{8\pi k_2^2}(3k_1^2 k_2^2-e^{-2k_1 t}), \tag{28}$$

$$\rho=-\frac{3\phi}{8\pi k_2^2}(k_1^2 k_2^2-e^{-2k_1 t}). \tag{29}$$

From (27)–(29), it can be seen that the weak energy conditions

$$\rho\ge 0, \qquad \rho+p\ge 0$$

and the strong energy conditions

$$\rho+p\ge 0, \qquad \rho+3p\ge 0$$

are identically satisfied if $k_1^2 k_2^2>1$.

In this case the deceleration parameter comes out to be negative ($b=-1$). This is consistent with the present observational results i.e. our model also suggest that the expansion of our universe is accelerating.

5 Solution in the Polynomial Form

Let $L(S)=\frac{1}{2k_3\sqrt{S}}$, where k_3 is constant.
With this form of $L(S)$, (23) on integration gives the exact solution

$$S(t)=(k_3t+k_4)^2, \tag{30}$$

where $k_4=kk_3$. With the help of (30), (20) yields the solution

$$\phi(t)=-c_3(k_3t+k_4)^{-m_3}-c_4(k_3t-k_4)^{-m_4}, \tag{31}$$

where $c_3>0$ and $c_4>0$ are constants of integration and

$$m_3=\frac{1}{2}(5-\sqrt{25+16\lambda}) \quad \text{and} \quad m_4=\frac{1}{2}(5+\sqrt{25+16\lambda}).$$

In this case also with suitable choice of λ within the limit ($|\lambda|\le 0.1$), we can get $m_3>0$ and $m_4>0$. This shows that scalar field ϕ always decreases with evolution of the universe. Further, by use of the solutions represented by (30) and (31), one can easily get expressions for pressure and energy density from (17) and (18) as

$$p=-\frac{\phi(t)}{8\pi}\left[\frac{8k_3^2(k_3t+k_4)^2-1}{(k_3+k_4)^2}\right], \tag{32}$$

$$\rho=-\frac{\phi(t)}{8\pi}\left[\frac{8k_3^2(k_3t+k_4)^2-1}{(k_3+k_4)^2}\right]. \tag{33}$$

Again, from (31)–(33), it can be seen that the weak energy conditions

$$\rho\ge 0, \qquad \rho+p\ge 0$$

and the strong energy conditions

$$\rho+p\ge 0, \qquad \rho+3p\ge 0$$

are identically satisfied if $k_3^2k_4^2>1$.

In this case the deceleration parameter is negative ($b=-\frac{1}{2}$). This is consistent with the present observational results i.e. our universe is accelerating.

6 Solution in the Sinusoidal Form

If we set $L(S)=\frac{1}{\sqrt{1-S^2}}$, then exact solution of (23) can be obtained as

$$S=\sin(t+k). \tag{34}$$

With the help of (34), (17), (18) and (20) yield the solution

$$p=-\frac{\phi(t)}{8\pi}\left[\frac{1-3\sin^2(t+k)-\sin^4(t+k)}{\sin^2(t+k)}\right], \tag{35}$$

$$\rho=\frac{3\phi(t)}{8\pi}. \tag{36}$$

$$\begin{aligned}\phi(t) = c_1\{\cot^2(t+k)+1\}^{\frac{3}{4}}\sin^{\frac{3}{2}}(t+k)\\ &\times \text{hypergeom}\left[\left\{\frac{3}{4}+\frac{\sqrt{9+8\lambda}}{4}, \frac{3}{4}-\frac{\sqrt{9+8\lambda}}{4}\right\}, \frac{1}{2}, \cos^2(t+k)\right]\\ &+c_2\{\cot^2(t+k)+1\}^{\frac{3}{4}}\{\cos(t+k)\sin^{\frac{3}{2}}(t+k)\}\\ &\times \text{hypergeom}\left[\left\{\frac{5}{4}-\frac{\sqrt{9+8\lambda}}{4}, \frac{5}{4}+\frac{\sqrt{9+8\lambda}}{4}\right\}, \frac{3}{2}, \cos^2(t+k)\right]. \end{aligned}\tag{37}$$

In this case the deceleration parameter is given by

$$b = \tan^2(t+k). \tag{38}$$

From above equation, we observe that the deceleration parameter is always positive. Since this is not consistent with recent observations, so don't seem to be viable for more consideration. So this case shall not be considered for study. This case is presented for the sake of mathematical completeness.

7 Conclusions

We have obtained new solutions for LRS Bianchi type I universe in Barber's second self creation theory. Many authors (References are already given in Introduction) have obtained solutions in Barber's second self creation theory but no body has considered the deceleration parameter to be dependent of time by now. This is the first solution in this alternative field theory where deceleration parameter is taken as time dependent. In past, the deceleration parameter q_0 was claimed to lie between 0 and 1 and thus it was claimed that the universe is decelerating. But present situation is hardly different. Observations [9, 34] of Type Ia Supernovae (SNe) allow to probe the expansion history of the universe. The main conclusion of these observations is that the universe is accelerating. In Sects. 4 and 5, we have obtained such types of models which suggest that the expansion of our universe is accelerating. Thus the presented solutions in this paper are free from singularity and the results are within the limit of observations.

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